NONEQUILIBRIUM IONIZATION IN A LAMINAR HYPERSONIC BOUNDARY LAYER

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UDC 533.601.155

Nonequilibrium flow of ionized air in a laminar boundary layer has been studied by several investigators [1-8], who have addressed a number of practical problems [2-5]. Nonequilibrium effects in a boundary layer manifest themselves particularly strongly in hypersonic flow of rarefied air. In this case the unperturbed flow possesses sufficient energy for dissociation and ionization, but the reactions proceed slowly, and their rate does not ensure a thermodynamically equilibrium composition. A boundary layer computation with nonequilibrium ionization was performed earlier, for the case of flow over axisymmetric bodies, assuming a low level of ionization [1-3]. With the latter condition one need not account for ionization in the energy equations, and also one can use the gasdynamic parameter profiles in the boundary layer from the zero-ionization problem to solve the nonequilibrium degree of ionization problem [1, 6]. However, in [1] greatly simplifying assumptions were made concerning the physical processes in the dissociating gas. For example, the authors did not account for recombination reactions and the mass flow of reacting molecular components, they assumed constant Schmidt number, and the dimensionless product of density and viscosity to be constant. In addition, they gave quite an inaccurate description of the processes for formation of atomic nitrogen and for cooling during dissociation. For these reasons it is of interest to obtain a solution to nonequilibrium ionization in the boundary layer with a more rigorous formulation of the problem.

In this paper we consider the problem of the laminar boundary layer of a multicomponent gas with a small degree of nonequilibrium ionization on the lateral surface of a body. We use a boundary condition for ions that is an improvement compared with that in [6, 9], which enables us to consider a small region where the quasineutral condition is violated near the wall (in contrast with what was done in [1-5]). The approximate analytical solution proposed makes it possible to rapidly calculate the maximum concentration of charged particles in the boundary layer when we have solutions for the neutral components. In applying the formulas obtained there is no difficulty in estimating the result of using these other models for the ionization processes in a multicomponent mixture and in explaining the discrepancies associated with the use of different reaction rate constants assumed by different authors [10-12].

1. Following references [1-6], we shall assume that the ionization is quite small, but enough to make a contribution to the momentum and energy equations. With this approach one can make full use of the calculated results of numerous investigators of the hypersonic boundary layer, in part of the distribution of velocity and temperature, and in the case of a dissociated mixture, of the distribution of neutral component concentrations. To do this one requires that

$$n_iV_i/ni \ll 1,$$

where V_i is the ionization energy; is the mean gas enthalpy per particle; n_i is the ion concentration; and n is the total concentration. Here we consider that the ionization is not so small as to invalidate the quasineutral condition of the boundary layer

$$d/\delta \ll 1 \tag{1.1}$$

(where d is the Debye radius, and δ is the displacement thickness). The latter inequality is a condition for formation of a thin layer of volume electric charge at the surface. Outside this layer we have the ambipolar diffusion region, and because of the quasineutral condition, $n_e \simeq n_i$, where n_e is the electron concentration.

As the gas approaches equilibrium the thickness of the Debye layer decreases, and, as estimate for air indicate, becomes comparable with the particle mean free path l. In the case

$$d/l \lesssim 1 \tag{1.2}$$

the boundary condition for the equation for conservation of charged particles will be written at the outer edge of the Langmuir layer, at a distance from the wall on the order of the mean free path. When Eq. (1.2) does not hold the approximate boundary condition is applied at the outer edge of the Debye layer.

Moscow. Translated from Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 1, pp. 49-56, January-February, 1979. Original article submitted January 30, 1978.

The wall is assumed to be non-heat-conducting, isothermal and quite cold, so that complete recombination of ions occurs there.

As a prerequisite of the problem, we introduce the hypothesis that we may neglect volume recombination in comparison with surface recombination. Numerical estimates for air indicate, e.g., that the charged particles are mainly formed in the high temperature layer, where there is insignificant recombination, and they then diffuse through relatively cold gas at the wall, where the rate of ionization–recombination processes in the volume is generally small compared with the surface rate.

From the experimental data of [5] and from an estimate it is assumed that the electron temperature is close to the gas temperature in the main part of the boundary layer.

We assume the existence of an electric field due to charge separation, which makes it necessary in the problem of flow over a nonconducting wall to take a coefficient, accounting for the induced electric field, as the effective diffusion coefficient in the equation for balance of a charged component of the mixture; this is the analogue of the well-known ambipolar diffusion coefficient in the very simple case of single-species ions [13].

Let ionization reactions of the form

$$X + Y \rightarrow XY^{+} + e \tag{1.3}$$

or

$$X + Y \rightarrow X^{+} + Y + e,$$

take place in the dissociation gas, and let them have a mass rate of

$$w_i = (m_i/m_X\, m_Y)\rho^2 c_X\, c_Y\, K_i$$

with coefficient K_i, depending on temperature. Here c_X, c_Y are the mass concentrations of the X and Y components, with particular masses m_X, m_Y; m_i is the ion mass; and ρ is the gas density.

For a homogeneous gas the ionization processes described by Eqs. (1.3) apply quite well, and for a dissociating mixture only approximately (due to the more complex ionization reactions). According to [14, 15], for air at high temperatures the relaxation time for ionization processes is greater than the relaxation time for dissociation.

A low level of ionization, associated mainly with the formation of NO^+ ions [16] occurs in problems with dissociating and ionized air, and conditions arise where the above assumptions hold*.

In the formulation of the problem adopted here the equations describing ionization are separated into a subsystem which may be solved after the dissociating boundary layer problem is solved. Here the ionization problem in the local similarity framework [17] gives linear differential equations, because the ionization rate is independent of charged particle concentration in the conditions considered.

We shall use the variables ξ and η in Lees' form [17]

$$\xi = \int_0^x (\rho\mu r^{2j})_w\, u_s dx, \qquad \eta = r_w^j u_s (2\xi)^{-1/2} \int_0^y \rho dy, \tag{1.4}$$

in which the ion conservation equation for the boundary layer under local similarity conditions takes the form

$$\frac{d}{d\eta}\left(\frac{N}{\mathrm{Sc}}\frac{dc_i^0}{d\eta}\right) + \varphi\frac{dc_i^0}{d\eta} + w_i^0 = 0, \tag{1.5}$$

where x and y are the coordinates along and normal to the body surface, respectively; r is the distance from the axis of symmetry to any point in the boundary layer; j = 0 for two-dimensional flow and j = 1 for axisymmetric flow; μ is the gas viscosity; Sc is the effective Schmidt number, and ρ is the density.

The dimensionless mass ion concentration, the particle formation rate, and the gas velocity and temperature, c_i^0, w_i^0, φ'_η, T^0 are associated with the dimensionless quantities as follows:

$$c_i^0 = \frac{c_i}{\zeta}, \quad w_i = \rho^0 c_X^0 c_Y^0 K_i^0, \quad \varphi'_\eta = \frac{u}{u_s}, \quad T^0 = \frac{T}{T_m}.$$

In addition,

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*The reaction forming the nitric oxide ion predominates in the shock layer up to a flight velocity of 9 km/sec [16]. Nonequilibrium flow in the boundary layer exists for $\zeta \lessgtr 1$.

$$\rho^0 = \frac{\rho}{\rho_m}, \quad c_X^0 = \frac{c_X}{c_{Xm}}, \quad c_Y^0 = \frac{c_Y}{c_{Ym}}, \quad K_i^0 = \frac{K_i}{K_{im}},$$

$$n_e^0 = \frac{n_e}{n_{em}}, \quad N = \frac{\rho\mu}{(\rho\mu)_w}, \quad \zeta = \tau_i^{-1} \frac{2\xi}{u_s} \frac{dx}{d\xi}, \quad \tau_i^{-1} = \frac{m_i}{m_X m_Y} \rho_m c_{Xm} c_{Ym} K_{im},$$

where ζ is the Damkeller number; and τ_i is the characteristic volume reaction time. The subscripts w, m, s denote parameters at the wall, at the maximum temperature, and at the outer edge of the boundary layer, respectively.

In the case of Eq. (1.1), if we neglect the influence of quasineutrality being perturbed, the boundary condition at the outer edge of the Debye layer will have a form similar to that assumed in [1-5]

$$c_i = 0$$

(it is assumed that the condition of Eq. (1.2) does not hold here).

The boundary condition in Eq. (1.2) associates the particle fluxes appearing in the gasdynamic equations with expressions obtained from gas-kinetic examination of transfer through a potential barrier at the outer edge of the Langmuir layer [6, 8, 18]:

$$-\frac{\mu}{\mathrm{Sc}} \frac{dc_i}{dy} = -\frac{1}{2} \rho c_i \left(\frac{8kT}{\pi m_i}\right)^{1/2} \tag{1.6}$$

(k is the Boltzmann constant). It is assumed, as was noted above, that there is complete recombination of ions at the wall, and this is reflected in the numerical coefficient on the right side of Eq. (1.6).

If we transfer to the variable η and add the condition at the outer edge of the boundary layer, for $\eta = 0$ we obtain the relations

$$c_i^0 = 0, \quad dc_i^0/d\eta - \varkappa c_i^0 = 0, \tag{1.7}$$

where

$$\varkappa = \frac{2\mathrm{Sc}_w}{(\mu r^j)_w u_s} \left(\frac{\xi k T_w}{\pi m_i}\right)^{1/2};$$

and at $\eta = \infty$

$$c_i^0 = c_{is}. \tag{1.8}$$

The linear boundary problem of Eqs. (1.5), (1.7) and (1.8) can be solved by quadratures (the zero subscript in the dimensionless quantities will be omitted from now on)

$$c_i(\eta) = [c_{is} + \psi(\infty)]\chi(\eta)/\chi(\infty) - \psi(\eta); \tag{1.9}$$

$$c_i(\eta) = \frac{c_{is} + \psi(\infty)}{1 + \varkappa\chi(\infty)} [1 + \varkappa\chi(\eta)] - \psi(\eta), \tag{1.10}$$

where

$$\psi(\eta) = \int_0^\eta E \int_0^t (w_i \mathrm{Sc}/EN)\, dv\, dt; \tag{1.11}$$

$$\chi(\eta) = \int_0^\eta E\, dt; \tag{1.12}$$

$$E = (N_w \mathrm{Sc}/N \mathrm{Sc}_w) \exp\left[-\int_0^\eta (\mathrm{Sc}\, \varphi/N)\, dt\right]. \tag{1.13}$$

The solution of Eq. (1.9) corresponds to the limiting case of $\varkappa \to \infty$ in Eq. (1.10).

Thus, the problem of determining the mass ion concentration profile in the boundary layer reduces to evaluating the quadratures Eqs. (1.11)-(1.13).

We note that the solution of Eqs. (1.9) and (1.10) is valid for an arbitrary law for variation of temperature and composition in the boundary layer. Therefore we can determine the ion concentration profile by using numerical solutions for a dissociating gas with varying Prandtl and Schmidt numbers. For example, the solutions of [2, 3, 7, 8] are known for dissociating air.

Taking into account the well-known law for pressure variation along a body surface, and knowing the dependence of viscosity, Schmidt number and the parameter N on temperature and pressure, we can carry out the quadratures of Eqs. (1.11)-(1.13) and find the ion concentration profile for a fixed value of the coordinate x. The profiles of velocity $\varphi'_\eta(\eta)$ and temperature $T(\eta)$ here are known from solving the problem with neutral components.

We note further that, having solved the problem for a dissociating gas, allowing for interaction between the boundary layer and the shock wave, as was done for a nonreacting gas on a flat plate in [19], we can also find the ion concentration profile in the problem when there is interaction and a given law p(x) for the pressure variation.

2. When the accuracy requirements for the quadrature of Eq. (1.11) are not so high, we can fall back on Eq. (1.12). Suitable expressions are obtained from the available data on dependence of ionization rate on the temperature and the shape of the temperature profile in a hypersonic boundary layer. It is known that at high Mach number, for a cold wall, the temperature profile has a pronounced maximum. Also, the ionization rate is exponential with temperature. Consequently, the main contribution to formation of charged particles comes from a narrow region of the boundary layer near the temperature maximum. This allows us to represent the ionization rate in terms of the δ function and to reduce the determination of mass ion concentration profile to evaluation of the quadrature of Eq. (1.12).

With constant Schmidt number across the boundary layer we obtain simple analytical formulas which enable us to make rapid approximate computations.* Estimates show that we can replace the exponent by the δ function, without introducing large errors.

We shall consider that [16]

$$K_i = T^\nu \exp\,[T_i(1 - 1/T)], \tag{2.1}$$

where

$$K_{im} = \sigma T_m^\nu \exp(-T_i), \quad T_i = D/T_m, \tag{2.2}$$

and the constants σ, ν, D are determined by the specific form of the reaction and the approximation made by various authors (for air see, e.g., [10-12]).

We shall approximate the temperature profile in a hypersonic boundary layer for a cooled wall in the maximum region and for the coordinates η_m by the function

$$T - 1 = -\omega(\eta - \eta_m)^2, \tag{2.3}$$

where ω is some constant.

After a series expansion of the exponent in Eq. (2.1), and the use of Eq. (2.3), we can write the ionization rate in terms of the δ function in the form

$$K_i = (\pi/\omega T_i)^{1/2}\delta(\eta - \eta_m). \tag{2.4}$$

By substituting Eq. (2.4) into Eq. (1.11), after the obvious transformations, we obtain

$$\psi(\eta) = \begin{cases} 0, & \eta \leqslant \eta_m, \\ \psi(\infty)\,[\chi(\eta) - \chi(\eta_m)]/[\chi(\infty) - \chi(\eta_m)], & \eta > \eta_m, \end{cases} \tag{2.5}$$

where

$$\psi(\infty) = (\pi/\omega T_i)^{1/2}[\mathrm{Sc}_m/N_m E(\eta_m)][\chi(\infty) - \chi(\eta_m)].$$

For Sc = const we obtain

$$E = [\varphi''/\varphi''(0)]^{\mathrm{Sc}}(N/N_w)^{\mathrm{Sc}-1},$$

$$\chi'_\eta(\eta) = \theta(\eta)/[\psi''(0)]^{\mathrm{Sc}}N_w^{\mathrm{Sc}-1}, \quad \theta(\eta) = \int_0^\eta [\varphi'']^{\mathrm{Sc}} N^{\mathrm{Sc}-1}dt,$$

$$\psi(\infty) = (\pi/\omega T_i)^{1/2}\,\mathrm{Sc}\,[\theta(\infty) - \theta(\eta_m)]/[\varphi''(\eta_m)N_m]^{\mathrm{Sc}}$$

and χ is replaced by θ in Eq. (2.5). Here the primes denote differentiation with respect to η.

In conclusion we write expressions for the parameter $\varkappa$ and the Damkeller number ξ for a sharp cone. From the first expression in Eq. (1.4), with $(\mu G/T)_w$ = const (G is the molecular weight), we obtain

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*The condition is not a limit of the problem, but is only regarded as one possible special case simplifying the final formulas, Eqs. (1.11), (1.12) and (2.5).

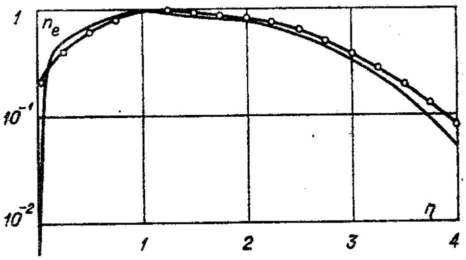

Fig. 1

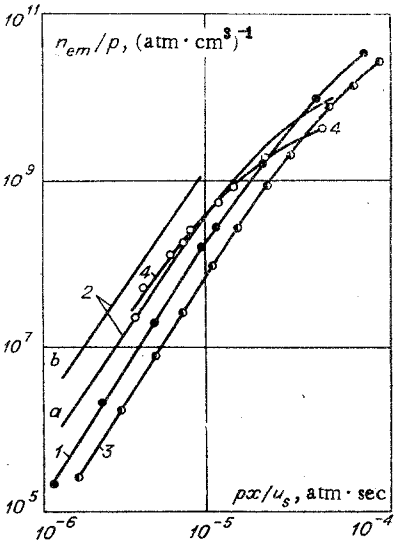

Fig. 2

$$\xi = \frac{d\xi}{dx}\frac{x}{3}. \tag{2.6}$$

Using Eqs. (2.2) and (2.6), after simple transformations we have

$$\varkappa = 2\mathrm{Sc}_w\left(\frac{m}{m_i}\frac{G_w}{3\pi\mu_w}\frac{px}{u_s}\right)^{1/2},$$

$$\zeta = \frac{2\sigma m_i m}{3km_X m_Y}c_{Xm}c_{Ym}G_m\frac{px}{u_s}T_m^{\nu-1}\exp(-T_i) \tag{2.7}$$

(m is the mass of the hydrogen atom).

Thus, knowing Sc, the profiles of φ and N, the parameters η_m, ω and $\varkappa$, we can find the distribution of the dimensionless mass ion concentration in the boundary layer, Eqs. (1.9) and (1.10), and, using Eq. (2.7), the absolute value for a fixed coordinate x.

3. As an example of the use of the analytical formulas we consider a sharp cone washed by air at constant pressure along the surface. To find the velocity profile we use the conventional temperature distribution [20]. In this case N is formally replaced by $N^* = (\rho\mu)^*/(\rho\mu)_w$, which we find from the value of the relative temperature T*, and the equation of motion is reduced to the Blasius equation for the function $\varphi/(N^*)^{1/2}$ with variable $\eta/(N^*)^{1/2}$. The dimensionless electron concentration and its maximum value for $\varkappa \gg 1$, $c_{is} \ll 1$ are found from the formulas

$$n_e = \theta(\infty)c_i/T\psi(\infty)\theta(\eta_m), \quad n_{em} = [\psi(\infty)\zeta pG_m/KT_m][m\theta(\eta_m)/m_i\theta(\infty)].$$

Our estimates show that the effective Schmidt number for dissociating air does not correspond to the values adopted in [1-4] (the authors did not account for the phenomenon of charge exchange). A better approximation will be given by Sc = 0.75, which is used in the calculations here.

Figure 1 shows the dimensionless electron concentration profile in the boundary layer (solid line) for the case of Eq. (1.2) with $px/u_s = 10^{-3}$ atm · sec, $N^* = 1$, $c_{is} \ll 1$, and, for comparison, the result obtained by the finite difference method of [2] (points). Both calculations refer to a cone semivertex angle of 10°, $u_s = 6585$ m/sec and $T_w = 1000$°K. Since, for a given cone semivertex angle and velocity of the incident flow, the temperature profiles vary only slightly [2, 3] with variation of the binary similarity parameter px/u_s [21], in the calculations we used the temperature distribution near the cone vertex for p = 0.247 atm [2]. In the example provided for comparison we also assumed the dimensionless concentration profile to be independent of the binary similarity parameter. We note that the behavior of the concentration for $\eta < 0.1$ was obtained here because of the improved boundary conditions. The agreement of the profile through the boundary layer with the result of accurate numerical calculations is fully satisfactory.

By extrapolating the maximum concentrations of the atomic components, obtained in [2] by the approximate method of [1], for our value of the binary similarity parameter, it may be shown that the condition of Eq. (1.2) is satisfied for p < 0.5 atm.

Figure 2 shows the results of calculated maximum electron concentration in the boundary layer for the case of Eq. (1.1) (curve 1). The parameter ω, which varies slightly with the binary similarity parameter [2, 3], was assumed to be equal to the value near the cone apex, and the maximum temperature and concentration of the atomic components were taken from [2]. Data were used relating to a constant rate for the ionization reaction $N+O \rightarrow NO^{+}+e$, as given in [10]. Figure 2 also shows results obtained using the approximate theory [1] (curve 2, a) $T_m = 4580°K$, b) $T_m = 4700°K$), by the finite difference method of [2] (curve 3) and by the method of integral relations of [3] (curve 4). We note that the results obtained here agree better with the exact numerical calculations of [2, 3] than with those determined by the approximate method of [1], although these do not require the use of a computer.

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